DEVELOPMENTS OF A CHILD'S FRACTION CONCEPTS WITH THE HELP OF PSYCHOLOGICAL TOOLS: A VYGOTSKY'S CULTURAL-HISTORICAL PERSPECTIVE

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This paper aims to interpret Vygotsky's abstract theory in a concrete mathematical context. Based on cultural-historical perspective Vygotsky stresses the importance of psychological tools in the development of human behavior. Through the interviewing and observing fraction lessons the researcher draws two conclusions: Both of the learning material and the fraction symbols function as psychological tools but they have difference in some levels; Kanako, a third grader, developed concepts of equivalent fractions mediated by fraction signs in a class, but it is not a real concept.

VYGOTSKY'S CULTULRAL-HISTORICAL PERSPECTIVE

According to Van der Veer and Valsiner (1991), Vygotsky's cultural-historical theory aimed at exploring where mental processes originated from and how they developed. In fact, Vygotsky and Luria (1930/1993) placed a great emphasis on "historical development" of human behavior, not only on "biological evolution" and "childhood development" (p.81). Based on four comparisons between behaviors of lower and higher forms such as between anthropoid apes and human beings (Van der Veer & Valsiner, 1991), Vygotsky and Luria (1930/1993) drew the following conclusions in reference to works of Köhler, Bühler, Engels, Lévy-Bruhl and so on (Yoshida, 2004a).

As regards nature, chimpanzees purely *use nature* with no intention, using tools as auxiliary. To make and use tools are inessential to survival for them. Using tools of labor, on the other hand, human beings *control nature* in accordance with their ends and plans. Hence these tools are essential to living for human.

Furthermore, regarding with psychological development, nonverbal communication and thought describe chimpanzees' behaviors whereas human beings invent artificial signs and behave relying on such signs and speech. This means that human beings *control their behavior itself using signs.* In short, Vygotsky recognized that the human ability to control behavior through sign systems was the key difference between anthropoid apes and humans.

Incidentally, what does it mean to control human behavior through sign systems? Vygotsky and Luria (1930/1993) illustrated it with comparisons between behaviors of *natural* people and *cultural* people. According to Roth, for instance, messengers of the North Queensland aborigines delivered a song repeating it from memory even though it took five nights to finish (as cited in Lévy-Bruhl, 1910/1966, p.94).

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Vygotsky gave an explanation of this as eidetic memory – an undifferentiated whole consisting of perception and memory – which people do *not control* but merely *use*.

Along with historical development of human beings, mnemotechnical aids became popular (Vygotsky & Luria, 1930/1993). For example, knot-based mnemotechnical systems for memorizing, or "quipu" were used to record important events or results of counting the number of animals (cf. p.104). Likewise, in Okinawa islands of Japan officers used to tie and interpret knots in a rope when collecting taxes (Ifrah, 1981/1988). And finally, human invented sings and letters for writing.

In conclusion, as sign systems developed, humans started to keep records with the help of mnemotechnical aids of knots and signs. In other words, human beings were freed from enormous amount of memories. As a result, it enabled humans to think abstractly, hypothetically, and logically (Vygotsky & Luria, 1930/1993). And this means that human beings control their behavior with the help of artificial signs.

PSYCHOLOGICAL TOOLS

> We have many reasons to assume that the cultural development consists in mastering methods of behavior which are based on the use of signs as a means of accomplishing any particular psychological operation. (Vygotsky, 1929, p.415)

This Vygotsky's description represents that self-control over behavior through sign systems is "the essence of the cultural development of man's behavior" (Vygotsky & Luria, 1930/1993, p.77). In this context *culture* has a special meaning for Vygotsky. As Van der Veer and Valsiner (1991) pointed out, using Barash's distinction of a cultural evolution, Vygotsky identified culture as sign systems – writing systems, counting systems, and language.

Such sign systems – the key to a cultural-historical development of human behavior – are explained as psychological tools by Vygotsky. Psychological tools are artificial instruments directed toward control over human behavior and they are the products of historical development of human behavior (Vygotsky, 1930/1997).

While natural memorization produces a direct associative connection A–B between stimuli A and B (see Figure 1), a psychological tool X makes a new path from A → X →B (Vygotsky, 1929, 1930/1997). At this time, X can play the *role of the object* which an act of behavior (ex. to memorize, to choose) for problem solving is directed toward and the *role of a means* of the psychological operations (ex. memorizing, comparing) to solve the problem.

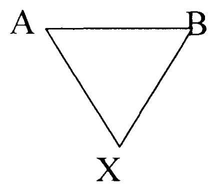

Figure 1: Schematic Triangle (Vygotsky, 1930/1997)

Moreover, psychological tools are differentiated from technical tools because the technical tools change the object itself while the psychological tools effect no change in the object but influence human behavior or mind.

This could be explained using the example of the officers in Okinawa islands shown in the above as follows: To record taxes the officers tie knots and to report the taxes they interpret the knots, and here the knots are the object of their acts. In addition, the knots play the role of a means that enables the officers to reach their objectives. Furthermore, the knots do not change the rope but change the officers' behavior.

Although the discussion given above is important theoretically, for mathematics educators it is more important to reinterpret it in a context of a real world on mathematics education. Thus, the researcher presents the following surveys and notes on lesson observations to consider development of a child's fraction concepts with the help of psychological tools.

A SERIES OF RESEARCHES ON KANAKO'S FRACTION CONCEPTS

Survey 1 and survey 2 by interviewing: Before and after fraction lessons

The surveys 1 and 2 for six third graders were conducted respectively in January and March 2000 (Yoshida, 2000b, 2001, 2002). The purposes of the surveys were to clarify children's *everyday concepts* of fractions and to identify how children's concepts develop before and after fraction lessons.

Kanako, one of the children, solved each problem first by herself while the researcher was observing her problem solving activity, and then was interviewed. The problems given in the surveys before and after the fraction lessons were almost the same, but only the latter survey included a number line problem.

The first problem was to make a classification and a characterization. In fact, in the survey 1 Kanako classified seven figures and sentences into two gropes according to features that they have in common, and named the groups "1 out of 3" and "2 out of 6."

In the survey 2, after learning fractions in classes, she classified them into two groups at first and named them "1/3" and "2/6." After interviewing her about the idea, the researcher asked if it would be possible to reduce the number of groups she classified into.

Researcher: ... What do you think if you can reduce the number of the groups you categorized into?

... (omission of some sentences by several people)

Kanako: I think it is OK to put the two groups together in one.

Researcher: Why do you think so?

Kanako: Well, because when 1/3 is marked with an additional scale, it turns out 2/6. Again, it gets bigger and bigger. ... why I separated this (group of 1/3) from that (group of 2/6) is because the numbers for dividing were different from one another. ...

Researcher: What kind of name do you give to the new group you made?

...

Kanako: a group of '1/3 transformed into 2/6' and '2/6'

The second problem was to draw a picture showing "one fourth" and to describe a meaning of "one half" with words. Kanako gave Figures 2 and 3 respectively before and after the fraction lessons.

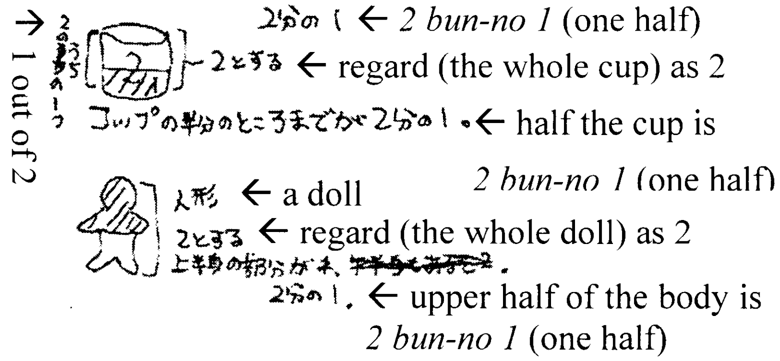

Figure 2: One half as "1 out of 2."

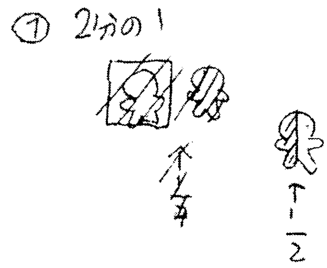

Figure 3: One half as "half a whole."

The third problem was to mark 4/10m, 1/2m, and 2/5m on a number line. Rika, one of the subjects in the surveys, gave an incorrect answer (see Appendix A) while Kanako gave a correct one (see Appendix B).

Observation on fraction lessons

A series of five fraction lessons for 39 third graders, including the six subjects in the surveys, were observed on March 1 – 7, 2000 in Hiroshima, Japan (Yoshida, 2002). Because of the official curriculum guidelines of that time, it was the first time for them to take fraction classes at school. A teacher specialized in mathematics set the following situations where children could learn fractions appropriately, according to his teaching experience.

In the first lesson, the teacher cut a piece of pink ribbon in two in front of the children and told them that the longer ribbon was 1m long. He asked how long the shorter one was. Through the lesson, they found out that triple of the length of the shorter ribbon was equivalent to that of the longer one. Moreover, a child raised a question; What would you do if the shorter ribbon did not correspond to the longer one entirely?

Therefore, in the second lesson the teacher asked the children to find out the length of a piece of new blue ribbon (45cm long) comparing to the longer pink ribbon (1m long) given in the first lesson. Through this lesson, the children realized that in this case it was the best way to fold the pink ribbon graduated in 1m. In short, they changed the benchmark for comparing from the ribbon in which the length was unknown to the ribbon with 1m-length.

In the next lesson, the teacher gave glass-shaped folding papers and asked, "This is a liter glass. How much juice is left in the glass?" Since the children had lots of experience of folding in the previous lesson, they started to solve this problem by folding the papers in some ways (see Figure 4). Through the folding activities, they found out that they could tell the amount of the juice in some ways depending on how many times they folded the paper. That means, some children gave the answer "2 out of 5 parts" while the other did "4 out of 10." After that, the teacher introduced a sign of fractions such as 2/5 and 4/10.

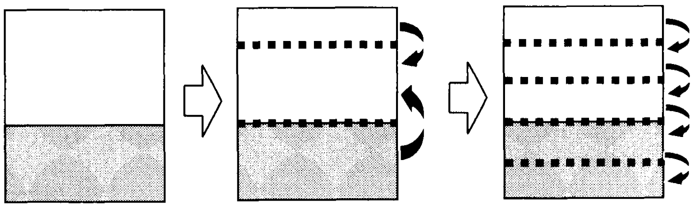
Figure 4: Glass-shaped folding papers.

The fourth lesson started with the same problem, but the amount of juice shaded on a glass-shaped folding paper was 3/5 liter. The focus of the children's interests changed from how to fold in the previous class to how to express the amount of the juice in this lesson, i.e. 6/10 l vs. 6/10 dl. Through an in-depth discussion on the issue, they achieved a consensus that it could be represented as 6/10 l and 3/5 l. The following conversation took place shortly after that.

Kazuo: Mr., you can make it more, endlessly.

... (omission of some sentences by several people)

Kanako: Um, I got started with 3/5, and then, 6/10, 9/15, ...

Teacher: 9/15? Wait. Just a second. I'm going to write them down here on the blackboard. 3/5, 9/15,

...

Teacher: 12/20. Ha! Ha! [while he is writing it down.]

Kanako: 15/25, 18/30, ...

At the end, Kanako notified that she gained 300/500, and so the other children looked into her notebook (see Appendix C) surrounding her desk.

DISCUSSION

The research suggests two findings as follows based on Vygotsky's theory.

First, we could regard both of the glass-shaped folding papers in the third lesson and fraction signs introduced in the fourth lesson as psychological tools to recognize the system of equivalent fractions, because both of them play the *role of a means* by which Kanako and the other children produced the idea of equivalent fractions as well as play the *role of the object* of their acts such as folding papers (to find out the amount of juice) and making fractions (to express the amount of juice in a variety of ways).

However, it is possible to distinguish the levels of the glass-shaped folding papers and fraction signs one another. That means the children and the officer who collected taxes depend on the concrete contexts using the glass-shaped folding papers and the knots, so that they cannot think and cannot solve their problems without the tools. On the other hand, after the teacher introduced the signs of fractions, Kanako developed expressions of equivalent fractions on her own motive without any concrete materials and not based on any practical situations. In short, the fraction signs as psychological tools could lead Kanako to a general, hypothetical, and/or abstract thinking and the ability of planning for the future. This corresponds with the following descriptions.

> With the aid of speech the child for the first time proves able to the mastering of its own behaviour, relating to itself as to another being, regarding itself as an object. Speech helps the child to master this object through the preliminary organization and planning of its own acts of behaviour. (Vygotsky & Luria, 1930/1994, p.111)

Second, we could conclude that Kanako's fraction concepts developed with the aid of psychological tools, i.e. fraction signs, through the lessons. For example, she modified her views on "one half" from "1 out of 2" (see Figure 2) to "half a whole" (see Figure 3). As Rika gave a wrong idea in the number line problem (see Appendix A), the idea of "1 out of 2" corresponding to each number of numerator and denominator for 1/2 is one of *everyday concepts* for fractions and causes the difficulty of learning fractions (cf. Yoshida, 2002, 2004b).

In addition, Kanako showed a remarkable development of fraction concepts in which she produced equivalent fractions from 3/5 to 300/500 during a class. However, the products made by Kanako (i.e. Appendix C) should be investigated with special attention because those equivalent fractions were probably made by adding 3 to the numerator and adding 5 to the denominator like $15/25 = (15+3)/(25+5) = 18/30$, instead of multiplying the numerator and denominator of 3/5 by 6, taking her age (or her ability) and the hours of the lesson into account.

Such Kanako's thinking for equivalent fractions are regarded as *pseudoconcept* (Vygotsky, 1934/1987). In appearance a pseudoconcept and a real concept look alike, yet in reality a pseudoconcept is one of thinking in *complexes*, and besides it is in the highest level among five different types of complex (cf. Yoshida, 2000a). Sierpinska (1993) gives an example of the pseudoconcept in mathematics as follows: Children may select every triangle in similar manner to adults; however, it is based on the physical appearance of triangles and not based on a definition of a triangle.

As Berger (2005) describes, pseudoconcepts can lead "the transition from complexes to concepts" (p.158); in addition to this, children can communicate with adults and other advanced people because of pseudoconcepts. Therefore, it is regarded that a pseudoconcept takes an important role when children's concepts or thinking develop. In fact, the teacher communicated with Kanako in the class as if she could have understood equivalent fractions properly. Yet the results of the first problem in the surveys 1 and 2 showed that Kanako's thinking of equivalent fractions was not enough to reach a real concept because Kanako combined the group of 1/3 and 2/6 only after the researcher asked if it would be possible.

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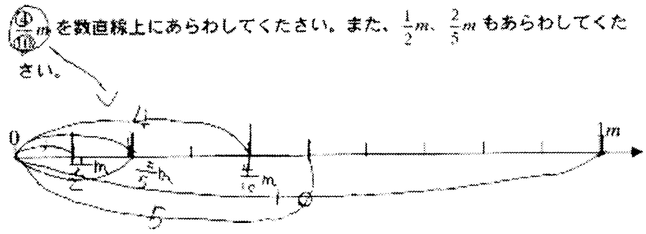
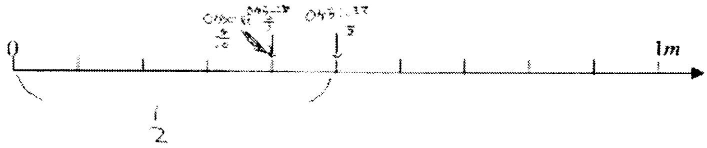
Appendix A: Rika's number line problem solving.



Appendix B: Kanako's number line problem solving.



Appendix C: Kanako created fractions starting with 3/5, on her own motive.